SOLUCIONES 1

1 $\int \frac{1}{x^2\ \sqrt[5]{x^2}}dx$

$$\int \frac{1}{x^2\ \sqrt[5]{x^2}}dx = \int x^{-2}x^{\frac{-2}{5}}dx = \int x^{\frac{-12}{5}}dx = \frac{x^{\frac{-12}{5}+1}}{\frac{-12}{5}+1} + C =$$

$$= \frac{x^{\frac{-7}{5}}}{\frac{-7}{5}} + C = -\frac{5}{7\sqrt[5]{x^7}} + C$$

2 $\int (x+2)^3\,dx$

$$\int (x+2)^3\,dx = \frac{1}{4}(x+2)^4 + C$$

3 $\int (2x+1)(x^2+x+1)dx$

$$\int (2x+1)(x^2+x+1)dx = \frac{1}{2}(x^2+x+1)^2 + C$$

4 $\int \frac{x+1}{\sqrt[3]{x^2+2x+7}}dx$

$$\int \frac{x+1}{\sqrt[3]{x^2+2x+7}}dx = \frac{1}{2}\int (2x+2)(x^2+2x+7)^{\frac{-1}{3}}dx =$$

$$= \frac{1}{2}\int \frac{(x^2+2x+7)^{\frac{2}{3}}}{\frac{2}{3}}dx = \frac{3}{4}\sqrt[3]{(x^2+2x+7)^2} + C$$

5 $\int sen\,2x\ \cos 2x\,dx$

$$\int sen\,2x\ \cos 2x\,dx = \frac{1}{2}\int sen\,2x\ \cos 2x \cdot 2dx = \frac{1}{4} sen^2\,2x\ + C$$

6 $\int \operatorname{sen}^4 x \cos x\,dx =$

$$\int \operatorname{sen}^4 x \cos x\,dx = \frac{1}{5}\operatorname{sen}^5 x + C$$

7 $\int \operatorname{tg}^2 x\ \sec^2 x\,dx$

$$\int \operatorname{tg}^2 x\ \sec^2 x\,dx = \frac{1}{3}\operatorname{tg}^3 x + C$$

8 $\int \frac{arc\ tg\ x}{1+x^2}dx$

$$\int \frac{arc\ tg\ x}{1+x^2}dx = \frac{1}{2}(arc\ tg\ x)^2\ + C$$

SOLUCIONES 2

1 $\int \frac{2x}{1+x^2}dx$

$$\int \frac{2x}{1+x^2}dx = \ln\left(1+x^2\right) + C$$

2 $\int \operatorname{tg} x\,dx$

$$\int tg\,x\,dx = \int \frac{sen\ x}{\cos x}dx = -\int \frac{-sen\ x}{\cos x}dx = -\ln\cos x\ + C$$

3 $\int \frac{5^{3x}}{5^{3x}+7}dx$

$$\int \frac{5^{3x}}{5^{3x}+7}dx = \frac{1}{3}\frac{1}{\ln 5}\int \frac{3\cdot 5^{3x}\ln 5}{5^{3x}+7}dx = \frac{1}{3\ln 5}\ \ln\left(5^{3x}+7\right)\ + C$$

4 $\int \frac{1}{x \ln x} dx$

$$\int \frac{1}{x \ln x} dx = \int \frac{\frac{1}{x}}{\ln x} dx = \ln(\ln x) + C$$

5 $\int \frac{1}{\cos^2 x \, tgx} dx$

$$\int \frac{1}{\cos^2 x \, tgx} dx = \int \frac{\frac{1}{\cos^2 x}}{tgx} dx = \ln(tgx) + C$$

6 $\int \frac{x+1}{x} dx$

$$\int \frac{x+1}{x} dx = \int \left(1 + \frac{1}{x}\right) dx = x + \ln x + C$$

7 $\int \frac{x+1}{x-5} dx$

$$\int \frac{x+1}{x-5} dx = \int \frac{x+1-5+5}{x-5} dx = \int \frac{x-5}{x-5} dx + \int \frac{6}{x-5} dx =$$

$$= x + 6\ln(x-5) + C$$

8 $\int \frac{3x^3+5x}{x^2+1} dx$

$3x^3 + 5x \quad \lfloor x^2 + 1$

$\boxed{2x} \quad 3x$

$$\int \frac{3x^3+5x}{x^2+1} dx = \int \left(3x + \frac{2x}{x^2+1}\right) dx = \frac{3}{2}x^2 + \ln(x^2+1) + C$$

SOLUCIONES 3

1 $\int e^{2x+2}\,dx$

$$\int e^{2x+2}\,dx = \frac{1}{2}e^{2x+2} + C$$

2 $\int 5^x\,dx$

$$\int 5^x\,dx = \frac{5^x}{\ln 5}$$

3 $\int 2^x\,5^x\,dx$

$$\int 2^x\,5^x\,dx = \int 10^x\,dx = \frac{10^x}{\ln 10} + C$$

4 $\int 8^{3x+1}\,dx$

$$\int 8^{3x+1}\,dx = \frac{1}{3}\int 8^{3x+1}\,3\,dx = \frac{1}{3\ln 8}8^{3x+1} + C$$

5 $\int \frac{e^{\ln x}}{x}\,dx$

$$\int \frac{e^{\ln x}}{x}\,dx = \int \frac{1}{x}\,e^{\ln x}\,dx = e^{\ln x} + C$$

6 $\int e^{\operatorname{sen} x}\cos x\,dx$

$$\int e^{\operatorname{sen} x}\cos x\,dx = e^{\operatorname{sen} x} + C$$

7 $\int \frac{e^{\operatorname{arc\,sen} x}}{\sqrt{1-x^2}}\,dx$

$$\int \frac{e^{\text{arc sen} x}}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{1-x^2}} e^{\text{arc sen} x} dx = e^{\text{arc sen} x} + C$$

SOLUCIONES 4

1 $\int (3 - \text{sen}\, x)dx$

$$\int (3 - \text{sen}\, x)dx = 3x + \cos x$$

2 $\int sen\,(3x+5)dx$

$$\int sen\,(3x+5)dx = \frac{1}{3}\int sen\,(3x+5)3\,dx = -\frac{1}{3}\cos(3x+5) + C$$

3 $\int (x+1)\ \text{sen}\left(x^2+2x+3\right)dx$

$$\int (x+1)\ sen\left(x^2+2x+3\right)dx = \frac{1}{2}\int (2x+2)\ sen\left(x^2+2x+3\right)dx =$$

$$= -\frac{1}{2}\cos\left(x^2+2x+3\right) + C$$

4 $\int e^x \text{sen}\, e^x\, dx$

$$\int e^x\, sen\, e^x\, dx = -\cos e^x$$

5 $\int \text{sen}\, 2x\ dx$

$$\int \text{sen}\, 2x\ dx = \frac{1}{2}\int \text{sen}\, 2x \cdot 2\,dx = -\frac{1}{2}\cos 2x + C$$

6 $\int sen^2\, 2x dx$

$$\int sen^2 2x dx = \int \frac{1-\cos 4x}{2} dx = \frac{1}{2}x - \frac{1}{8} sen 4x + C$$

7 $\int \operatorname{sen}^3 x\, dx$

$$\int \operatorname{sen}^3 x\, dx = \int \operatorname{sen}^2 x \ \operatorname{sen} x\, dx = \int \left(1-\cos^2 x\right)\operatorname{sen} x\, dx =$$

$$\int \left(\operatorname{sen} x - \cos^2 x \ \operatorname{sen} x\right) dx = -\cos x + \frac{1}{3}\cos^3 x + C$$

SOLUCIONES 5

1 $\int (2x + \cos x)\, dx$

$$\int (2x + \cos x)\, dx = x^2 + \operatorname{sen} x$$

2 $\int \cos(2x+5)\, dx$

$$\int \cos(2x+5)\, dx = \frac{1}{2}\operatorname{sen}(2x+5) + C$$

3 $\int (x+1)\cos\left(x^2+2x+1\right) dx$

$$\int (x+1)\cos\left(x^2+2x+1\right) dx = \frac{1}{2}\int (2x+2)\cos\left(x^2+2x+1\right) dx =$$

$$= sen\left(x^2+2x+1\right) + C$$

4 $\int \frac{\cos(\ln x)}{x} dx$

$$\int \frac{\cos(\ln x)}{x} dx = \int \cos(\ln x)\frac{1}{x} dx = \operatorname{sen}(\ln x) + C$$

5 $\int \cos^2 x\, dx$

$$\int \cos^2 x\, dx = \int \left(\sqrt{\frac{1+\cos 2x}{2}}\right)^2 dx = \int \frac{1+\cos 2x}{2}\, dx =$$

$$\frac{1}{2}\int (1+\cos 2x)dx = \frac{1}{2}\left(x+\frac{1}{2}\text{sen}2x\right) + C = \frac{1}{2}x + \frac{1}{4}\text{sen}2x + C$$

6 $\int \cos^3 3x\, dx$

$$\int \cos^3 3x\, dx = \int \cos^2 3x \cos 3x\, dx = \int \left(1-\text{sen}^2 3x\right)\cos 3x\; dx =$$

$$= \int \cos 3x\, dx - \int \text{sen}^2 3x \cos 3x\, dx = \frac{1}{3}\text{sen}\, 3x - \frac{1}{9}\text{sen}^3 3x + C$$

SOLUCIONES 6

1 $\int \frac{5}{\cos^2 x} dx$

$$\int \frac{5}{\cos^2 x} dx = 5 tg x + C$$

2 $\int \left(3+3 tg^2 x\right)dx$

$$\int \left(3+3tg^2 x\right)dx = 3\int \left(1+tg^2 x\right)dx = 3 tg\, x + C$$

3 $\int \sec^2(5x+3)dx$

$$\int \sec^2(5x+3)dx = \frac{1}{5}\int \sec^2(5x+3)5dx = \frac{1}{5} tg\,(5x+3) + C$$

4 $\int \sec^4 x\, dx$

$$\int \sec^4 x\,dx = \int \sec^2 x\ \sec^2 x\,dx = \int \left(1+tg^2 x\right) \sec^2 x\,dx =$$

$$\int \left(\sec^2 x + \sec^2 x\ tg^2 x\right)dx = tg\,x + \frac{1}{3}tg^3 x\ +\ C$$

5 $$\int \left(3+3\cot g^2 x\right)dx$$

$$\int \left(3+3\cot g^2 x\right)dx = 3\int \left(1+\cot g^2 x\right)dx = -3\cot g\,x + C$$

6 $$\int tg^2 x\ dx$$

$$\int tg^2 x\ dx = \int \left(1+tg^2 x - 1\right)dx = \int \left(1+tg^2 x\right)dx - \int dx = tg\,x - x + C$$

SOLUCIONES 7

1 $$\int \operatorname{sen} 3x \cos 2x\,dx$$

$$\operatorname{sen} A + \operatorname{sen} B = 2\operatorname{sen}\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\begin{cases} \dfrac{A+B}{2} = 3x \\ \dfrac{A-B}{2} = 2x \end{cases} \qquad A = 5x \qquad B = x$$

$$\int \operatorname{sen} 3x \cos 2x\,dx = \frac{1}{2}\int 2\operatorname{sen} 3x \cos 2x\,dx =$$

$$= \frac{1}{2}\int \left(\operatorname{sen} 5x + \operatorname{sen} x\right)dx = \frac{1}{2}\left(-\frac{\cos 5x}{5} - \cos x\right) + C$$

2 $$\int \frac{dx}{\operatorname{sen}^2 x \cos^2 x}$$

$$\int \frac{dx}{\operatorname{sen}^2 x \cos^2 x} = \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen}^2 x \cos^2 x} dx =$$

$$\int \frac{\operatorname{sen}^2 x}{\operatorname{sen}^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\operatorname{sen}^2 x \cos^2 x} dx =$$

$$= \int \frac{dx}{\cos^2 x} + \int \frac{dx}{\operatorname{sen}^2 x} = \operatorname{tg} x - \operatorname{cotg} x + C$$

3 $$\int \sqrt{\frac{1+x}{1-x}} dx$$

$$\int \sqrt{\frac{1+x}{1-x}} dx = \int \frac{1+x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx =$$

$$= \int \frac{1}{\sqrt{1-x^2}} dx - \frac{1}{2} \int (-2x)\left(1-x^2\right)^{\frac{-1}{2}} dx =$$

$$= arc\ sen\, x - \sqrt{1-x^2} + C$$

4 $$\int \frac{1-\cos x}{1+\cos x} dx$$

$$\int \frac{1-\cos x}{1+\cos x} dx = \int \frac{\left(1-\cos x\right)^2}{1-\cos^2 x} dx = \int \frac{1-2\cos x + \cos^2 x}{sen^2 x} dx =$$

$$= \int \frac{1}{sen^2 x} dx - 2\int \cos x \; sen^{-2} x\, dx + \int \cot g^2 x\, dx =$$

$$= \int \frac{1}{sen^2 x} dx - 2\int \cos x \; sen^{-2} x\, dx + \int \left[\left(1 + \cot g^2 x\right) - 1\right] dx =$$

$$= -\cot g\, x + \frac{2}{sen\, x} - \cot g\, x - x + C =$$

$$= -2\cot g\, x + \frac{2}{sen\, x} - x + C$$

5 $\int \frac{1+sen\,x}{1-sen\,x}dx$

$$=\int \frac{(1+sen\,x)^2}{1-sen^2x}dx=\int \frac{1+2sen\,x+sen^2x}{\cos^2 x}dx=$$

$$=\int \frac{1}{\cos^2 x}dx+2\int \frac{senx}{\cos^2 x}dx+\int \frac{sen^2x}{\cos^2 x}dx=$$

$$=\int \frac{1}{\cos^2 x}dx-2\int (-senx)\cos^{-2} x\,dx+\int tg^2 x\,dx=$$

$$=\int \frac{1}{\cos^2 x}dx-2\int (-senx)\cos^{-2} x\,dx+\int\left[\left(1+tg^2 x\right)-1\right]dx=$$

$$=tg\,x+\frac{2}{\cos x}+tg\,x-x\ +\ C=$$

$$=2tg\,x+\frac{2}{\cos x}-x\ +\ C$$

SOLUCIONES 8

1.-

$$\int \frac{5}{x^2-4x+8}dx$$

$$\int \frac{5}{x^2-4x+8}dx=\int \frac{5}{x^2-4x+4+4}dx=\int \frac{5}{4+(x-2)^2}dx=$$

$$=\frac{5}{4}\int \frac{dx}{1+\left(\frac{x-2}{2}\right)^2}=\frac{5}{4}\cdot 2\int \frac{\frac{1}{2}}{1+\left(\frac{x-2}{2}\right)^2}dx=$$

$$= \frac{5}{2} arc\ tg\left(\frac{x-2}{2}\right) + C$$

2.-

$$\int \frac{2x+5}{\sqrt{9-x^2}}dx$$

$$\int \frac{2x+5}{\sqrt{9-x^2}}dx = \int \frac{2x}{\sqrt{9-x^2}}dx + \int \frac{5}{\sqrt{9-x^2}}dx$$

$$= -\int \left(9-x^2\right)^{-\frac{1}{2}}(-2x)dx + \frac{5}{3}\cdot 3\int \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}}dx =$$

$$= -\frac{\left(9-x^2\right)^{\frac{1}{2}}}{\frac{1}{2}} + 5\,\text{arc sen}\left(\frac{x}{3}\right) + C = -2\sqrt{9-x^2} + 5\,\text{arc sen}\left(\frac{x}{3}\right) + C$$

3.-

$$\int \frac{2^x}{\sqrt{1-4^x}}dx$$

$$\int \frac{2^x}{\sqrt{1-4^x}}dx = \int \frac{2^x}{\sqrt{1-\left(2^x\right)^2}}dx = \frac{1}{\ln 2}\int \frac{2^x \ln 2}{\sqrt{1-\left(2^x\right)^2}}dx =$$

$$= \frac{1}{\ln 2}\text{arc sen}\left(2^x\right) + C$$

4.-

$$\int \frac{x}{\sqrt{9-2x^4}}dx$$

$$\int \frac{x}{\sqrt{9-2x^4}}\,dx = \int \frac{x}{\sqrt{9\left[1-\left(\frac{\sqrt{2}\,x^2}{3}\right)^2\right]}}\,dx =$$

$$= \frac{1}{3}\frac{3}{2\sqrt{2}}\int \frac{\frac{2\sqrt{2}}{3}}{\sqrt{1-\left(\frac{\sqrt{2}\,x^2}{3}\right)^2}}\,dx = \frac{1}{2\sqrt{2}}\,\text{arc sen}\left(\frac{\sqrt{2}}{3}\,x^2\right) + C$$