

Ejercicios resueltos de integrales inmediatas

1

$$\int \cos x \sqrt{\operatorname{sen} x} dx$$

$$\int \cos x \sqrt{\operatorname{sen} x} dx = \int (\operatorname{sen} x)^{\frac{1}{2}} \cos x dx = \frac{(\operatorname{sen} x)^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2}{3} \operatorname{sen} x \sqrt{\operatorname{sen} x} + C$$

2

$$\int \sec^3 x \operatorname{tg} x dx$$

$$y = \sec x \quad y' = \sec x \operatorname{tg} x$$

$$\int \sec^2 x \sec x \operatorname{tg} x dx = \frac{1}{3} \sec^3 x + C$$

3

$$\int \frac{dx}{\sqrt{x} \cos^2 \sqrt{x}}$$

$$\int \frac{dx}{\sqrt{x} \cos^2 \sqrt{x}} = 2 \int \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x} \cos^2 \sqrt{x}} dx = 2 \operatorname{tg} \sqrt{x} + C$$

4

$$\int \frac{dx}{x(1+\ln x)^3}$$

$$\int \frac{dx}{x(1+\ln x)^3} = \int (1+\ln x)^{-3} \frac{1}{x} dx = -\frac{(1+\ln x)^{-2}}{2} + C =$$

5

$$\int \frac{\operatorname{sen}^2 x}{\cos^4 x} dx$$

$$\begin{aligned} \int \frac{\operatorname{sen}^2 x}{\cos^4 x} dx &= \int \frac{\operatorname{sen}^2 x}{\cos^2 x} \frac{1}{\cos^2 x} dx = \int \operatorname{tg}^2 x \frac{1}{\cos^2 x} dx = \\ &= \frac{1}{3} \operatorname{tg}^3 x + C \end{aligned}$$

6

$$\int \frac{\operatorname{sen} x + \operatorname{tg} x}{\cos x} dx$$

$$\begin{aligned} \int \frac{\operatorname{sen} x + \operatorname{tg} x}{\cos x} dx &= \int \frac{\operatorname{sen} x}{\cos x} dx + \int \frac{\operatorname{tg} x}{\cos x} dx = \\ &= \int \frac{\operatorname{sen} x}{\cos x} dx + \int \operatorname{tg} x \sec x dx = -\ln(\cos x) + \sec x + C \end{aligned}$$

7

$$\int \frac{dx}{\operatorname{sen} x \cos x}$$

$$\begin{aligned} \int \frac{dx}{\operatorname{sen} x \cos x} &= \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cos x} dx = \int \frac{\operatorname{sen} x}{\cos x} dx + \int \frac{\cos x}{\operatorname{sen} x} dx = \\ &= -\ln(\cos x) + \ln(\operatorname{sen} x) + C = \ln(\operatorname{tg} x) + C \end{aligned}$$

8

$$\int \sqrt{x}\sqrt{x} dx$$

$$\int \sqrt{x}\sqrt{x} dx = \int \sqrt{\sqrt{x^2 \cdot x}} dx = \int \sqrt[4]{x^3} dx = \int x^{\frac{3}{4}} dx = \int \frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1} dx =$$

9

$$\int \sqrt[3]{x} \sqrt{\frac{2}{x}} dx$$

$$\int \sqrt[3]{x} \sqrt{\frac{2}{x}} dx = \int \sqrt[3]{\frac{\sqrt{2x^2}}{x}} dx = \int \sqrt[6]{2x} dx = \sqrt[6]{2} \int x^{\frac{1}{6}} dx = \sqrt[6]{2} \int \frac{x^{\frac{1}{6}+1}}{\frac{1}{6}+1} dx =$$

$$= \frac{6}{7} \sqrt[6]{2} x^{\frac{7}{6}} + C = \frac{6}{7} \sqrt[6]{2} \sqrt[6]{x^7} + C = \frac{6}{7} x \sqrt[6]{2x} + C$$

10

$$\int \frac{\cos x}{\sqrt{\operatorname{sen}^3 x}} dx$$

$$\int \frac{\cos x}{\sqrt{\operatorname{sen}^3 x}} dx = \int \cos x \operatorname{sen}^{-\frac{3}{2}} x dx = \frac{\operatorname{sen}^{-\frac{1}{2}} x}{-\frac{1}{2}} + C = \frac{-2}{\sqrt{\operatorname{sen} x}} + C$$

11

$$\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x} dx$$

$$\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x} dx = - \int \frac{\cos x - \operatorname{sen} x}{\operatorname{sen} x + \cos x} dx =$$

$$= -\ln(\sin x + \cos x) + C$$

12

$$\int \frac{dx}{(1+x^2) \operatorname{arc} \operatorname{tg} x}$$

$$\int \frac{dx}{(1+x^2) \operatorname{arc} \operatorname{tg} x} = \int \frac{1}{\operatorname{arc} \operatorname{tg} x} \frac{1}{1+x^2} dx =$$

$$= \ln(\operatorname{arc} \operatorname{tg} x) + C$$

13

$$\int \frac{\sec^2 x}{1 + \operatorname{tg}^2 x} dx$$

$$\int \frac{\sec^2 x}{1 + \operatorname{tg}^2 x} dx = \operatorname{arc} \operatorname{tg}(\operatorname{tg} x) + C = x + C$$

14

$$\int \frac{(2 \ln x)^2}{4x} dx$$

$$\int \frac{(2 \ln x)^2}{4x} dx = \int \ln^2(x) \frac{1}{x} dx = \frac{1}{3} \ln^3 x + C$$

15

$$\int \frac{\ln x^2}{x} dx$$

$$\int \frac{\ln x^2}{x} dx = \int 2 \ln(x) \frac{1}{x} dx = \ln^2 x + C$$

16

$$\int \frac{dx}{\sqrt{x} \cos^2 \sqrt{x}}$$

$$\int \frac{dx}{\sqrt{x} \cos^2 \sqrt{x}} = 2 \int \frac{dx}{\cos^2 \sqrt{x}} \frac{1}{2\sqrt{x}} dx = 2 \operatorname{tg} \sqrt{x} + C$$

17

$$\int \frac{4^x + 5 \cdot 16^x}{1 + 16^x} dx$$

$$\int \frac{4^x + 5 \cdot 16^x}{1 + 16^x} dx = \int \frac{4^x}{1 + (4^x)^2} dx + 5 \int \frac{4^{2x}}{1 + 4^{2x}} dx =$$

$$= \frac{1}{\ln 4} \operatorname{arc} \operatorname{tg}(4^x) + \frac{5}{2 \ln 4} \ln(1 + 4^{2x}) + C$$

18

$$\int \frac{\operatorname{tg} \sqrt{x}}{\sqrt{x}} dx$$

$$-2 \int \frac{-\operatorname{sen} \sqrt{x}}{\cos \sqrt{x}} \frac{1}{2\sqrt{x}} dx = -2 \ln(\cos \sqrt{x}) + C$$

19

$$\int \frac{\sqrt{7 + 2 \operatorname{tg} x}}{\cos^2 x} dx$$

$$\int \frac{\sqrt{7 + 2 \operatorname{tg} x}}{\cos^2 x} dx = \frac{1}{2} \int (7 + 2 \operatorname{tg} x)^{\frac{1}{2}} \frac{2}{\cos^2 x} dx =$$

$$= \frac{1}{2} \frac{(7 + 2 \operatorname{tg} x)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3} (7 + 2 \operatorname{tg} x) \sqrt{7 + 2 \operatorname{tg} x} + C$$

$$\int \frac{dx}{x\sqrt{1-\ln^2 x}}$$

$$\int \frac{dx}{x\sqrt{1-\ln^2 x}} = \int \frac{\frac{1}{x}}{\sqrt{1-\ln^2 x}} dx = \text{arc sen}(\ln x) + C$$